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Transient piezothermoelasticity for a cylindrical composite panel composed of angle-ply and piezoelectric laminae

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Abstract

This paper is concerned with the theoretical treatment of transient piezothermoelastic problem is developed for a cylindrical composite panel composed of angle-ply laminae and piezoelectric material of crystal class mm2, subject to non-uniform heat supply in the circumferential direction. We obtain the exact solution for the two-dimensional temperature change in a transient state, and transient piezothermoelastic response of a simple supported cylindrical composite panel under the state of generalized plane deformation. As an example, numerical calculations are carried out for an angle-ply laminated composite panel made of alumina fiber reinforced aluminum composite, associated with a piezoelectric layer of a cadmium selenide solid. Some numerical results for temperature change, displacement, stress and electric potential distributions in a transient state are shown in figures. Furthermore, the influence of thickness of the angle-ply laminate is investigated.

Keywords: Piezothermoelasticity; Angle-ply; Piezoelectricity; Cylindrical composite panel; Generalized plane deformation; Transient state

1. Introduction

Recently, as the need for the material structures with new functions such as perception, decision and acknowledge increases, smart composite structures composed of piezoelectric materials have received attention. Since the piezoelectric materials have piezoelectric effects and inverse piezoelectric effects, they can be used in the smart composite structures as sensors and as actuators. A basic element of these smart composite structures is a laminated piezoelectric structure, and the analytical studies concerned with piezothermoelasticity were developed (Tauchert et al., 2000). Furthermore, one of the causes of damage in this laminated piezoelectric structure includes delamination. In order to evaluate this phenomenon, it is necessary to analyze the piezothermoelastic problems taking into account the transverse stress components. From the above concept, as a steady piezothermoelastic problem, the three-dimensional problem of

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rectangular laminated plate (Xu et al., 1995) or the problem for cylindrical bending of simply supported laminated infinitely long plate (Kapuria et al., 1997a) were analyzed exactly. And the axisymmetric piezothermoelastic problems of composite circular plates were investigated by Tauchert and Ashida (1999). As a transient piezothermoelastic problem, we recently analyzed exactly the three-dimensional problems of rectangular composite plate composed of cross-ply and piezoelectric laminae (Ootao and Tanigawa, 2000a), and functionally graded rectangular plate bonded to a piezoelectric plate (Ootao and Tanigawa, 2000b) taking into account all transverse stress components. However, these exact studies discuss the problems of plates.

On the other hand, the piezothermoelastic analyses of cylindrical panels and cylindrical shells with curvature are important as well as those of plate models. However, since their analytical treatments are very complex and difficult, there are few exact analysis concerned with the piezothermoelastic problems of laminated cylindrical panels and shells taking into account transverse stress components. For example, the axisymmetric piezothermoelastic problem of a cylindrical laminated shell was reported by Chen and Shen (1996). And the three-dimensional piezothermoelastic problems of cylindrical laminated shells were analyzed by Xu and Noor (1996) and Kapuria et al. (1997b), and the three-dimensional piezothermoelastic problem of cylindrical laminated panel was analyzed by Kapuria et al. (1997c). The piezothermoelastic solution for angle-ply laminated cylindrical panel under cylindrical bending was presented by Dumir et al. (1997). These papers, however, treated only the piezothermoelastic problems under the steady temperature distribution. To the author's knowledge, the exact analysis for a transient piezothermoelastic problem of laminated cylindrical panel composed of piezoelectric material has not been reported.

In the present article, we have treated exactly the transient piezothermoelastic problem of a simply supported cylindrical composite panel due to a non-uniform heat supply in the circumferential direction. It is assumed that the cylindrical composite panel is composed of angle-ply laminate and piezoelectric material of crystal class mm2.

2. Analysis

We consider an infinitely long, angle-ply laminated cylindrical panel to which a piezoelectric layer of crystal class mm2 is perfectly bonded, the length of the side in the circumferential direction of which is denoted by θ_0. The combined panel's inner and outer radii are designated c and b, respectively. Moreover, a is the coordinate of interface between the angle-ply laminate and the piezoelectric layer. Throughout this article, the quantities with subscripts $i = 2, \ldots, N+1$ and $i = 1$ denote those for ith layer of the angle-ply laminate and piezoelectric layer, respectively. It is assumed that each layer of the angle-ply laminate maintains the orthotropic material properties and the fiber direction in the ith layer is alternated with ply angle ϕ_i to the z-axis. It is assumed that the principal axes of the piezoelectric layer are parallel to the axes of the cylindrical coordinate, and the piezoelectric layer is poled in the radial direction.

2.1. Heat conduction problem

We assume that the laminated cylindrical panel is initially at zero temperature and is suddenly heated partially from the outer surface by surrounding media, the temperature of which is denoted by the function $T_b f_b(\theta)$. The relative heat transfer coefficients on the inner and outer surfaces of the combined panel are designated h_c and h_b, respectively. We assume that the edges of the combined panel are held at zero temperature. Then the temperature distribution shows a two-dimensional distribution in $r-\theta$ plane, and the transient heat conduction equation for the ith layer and the initial and thermal boundary conditions in dimensionless form are taken in the following forms:

$$\frac{\partial \overline{T}_i}{\partial \tau} = \bar{\kappa}_{ri}\left(\frac{\partial^2 \overline{T}_i}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \overline{T}_i}{\partial \rho}\right) + \frac{\bar{\kappa}_{\theta i}}{\rho^2}\frac{\partial^2 \overline{T}_i}{\partial \theta^2}; \quad i = 1\text{–}(N+1) \tag{1}$$

$$\tau = 0; \quad \overline{T}_i = 0; \qquad i = 1\text{–}(N+1) \tag{2}$$

$$\rho = \bar{c}; \quad \frac{\partial \overline{T}_1}{\partial \rho} - H_c \overline{T}_1 = 0 \tag{3}$$

$$\rho = R_i; \quad \overline{T}_i = \overline{T}_{i+1}; \qquad i = 1\text{–}N \tag{4}$$

$$\rho = R_i; \quad \bar{\lambda}_{ri}\frac{\partial \overline{T}_i}{\partial \rho} = \bar{\lambda}_{r,i+1}\frac{\partial \overline{T}_{i+1}}{\partial \rho}; \qquad i = 1\text{–}N \tag{5}$$

$$\rho = 1; \quad \frac{\partial \overline{T}_{N+1}}{\partial \rho} + H_b \overline{T}_{N+1} = H_b \overline{T}_b f_b(\theta) \tag{6}$$

$$\theta = 0,\ \theta_0; \quad \overline{T}_i = 0; \qquad i = 1\text{–}(N+1) \tag{7}$$

where

$$\bar{\kappa}_{ri} = \bar{\kappa}_{T_i}, \quad \bar{\kappa}_{\theta i} = \bar{\kappa}_{L_i}\sin^2\phi_i + \bar{\kappa}_{T_i}\cos^2\phi_i, \quad \bar{\lambda}_{ri} = \bar{\lambda}_{T_i}; \quad i = 2\text{–}(N+1) \tag{8}$$

$$R_1 = \bar{a} \tag{9}$$

In expressions (1)–(9), we have introduced the following dimensionless values:

$$(\overline{T}_i, \overline{T}_b) = \frac{(T_i, T_b)}{T_0}, \quad (\rho, R_i, \bar{a}, \bar{c}) = \frac{(r, r_i, a, c)}{b}, \quad (\bar{\kappa}_{ri}, \bar{\kappa}_{\theta i}, \bar{\kappa}_{L_i}, \bar{\kappa}_{T_i}) = \frac{(\kappa_{ri}, \kappa_{\theta i}, \kappa_{L_i}, \kappa_{T_i})}{\kappa_0},$$
$$(\bar{\lambda}_{ri}, \bar{\lambda}_{T_i}) = \frac{(\lambda_{ri}, \lambda_{T_i})}{\lambda_0}, \quad \tau = \frac{\kappa_0 t}{b^2}, \quad (H_b, H_c) = (h_b, h_c)b \tag{10}$$

where T_i is the temperature change of the ith layer; κ_{ri} and $\kappa_{\theta i}$ are thermal diffusivities in the r and θ directions, respectively; λ_{ri} is thermal conductivity in the r direction; t is time; and T_0, κ_0, and λ_0 are typical values of temperature, thermal diffusivity and thermal conductivity, respectively. In Eq. (8), the subscripts L and T denote the fiber and transverse directions, respectively. Moreover, r_i $(i = 1, 2, \ldots, N)$ are the coordinates of interface of the laminated cylindrical panel.

Introducing the finite sine transformation with respect to the variable θ and Laplace transformation with respect to the variable τ, the solution of Eq. (1) can be obtained so as to satisfy the conditions (2)–(7). This solution is shown as follows:

$$\overline{T}_i = \sum_{k=1}^{\infty} \overline{T}_{ik}(\rho, \tau)\sin q_k\theta; \quad i = 1\text{–}(N+1) \tag{11}$$

where

$$\overline{T}_{ik}(\rho, \tau) = \frac{2}{\theta_0}\left[\frac{1}{F}(\overline{A}'_i\rho^{\gamma_i} + \overline{B}'_i\rho^{-\gamma_i}) + \sum_{j=1}^{\infty}\frac{2\exp(-\mu_j^2\tau)}{\mu_j \Delta'(\mu_j)}\{\overline{A}_i J_{\gamma_i}(\beta_i\mu_j\rho) + \overline{B}_i Y_{\gamma_i}(\beta_i\mu_j\rho)\}\right] \tag{12}$$

where $J_\gamma()$ and $Y_\gamma()$ are the Bessel function of the first and second kind of order γ, respectively; Δ and F are the determinants of $2(N+1)\times 2(N+1)$ matrix $[a_{kl}]$ and $[e_{kl}]$, respectively; the coefficients $\overline{A}_i$ and $\overline{B}_i$ are defined as the determinants of the matrix similar to the coefficient matrix $[a_{kl}]$, in which the $(2i-1)$th column or 2ith column is replaced by the constant vector $\{c_k\}$, respectively; similarly, the coefficients $\overline{A}_i'$ and $\overline{B}_i'$ are defined as the determinants of the matrix similar to the coefficient matrix $[e_{kl}]$, in which the $(2i-1)$th column or 2ith column is replaced by the constant vector $\{c_k\}$, respectively. Furthermore, the non-zero elements a_{kl} and c_k of the coefficient matrix $[a_{kl}]$ and the constant vector $\{c_k\}$ are given as follows:

$$
\begin{aligned}
&a_{11} = \beta_1 \mu J_{\gamma_1+1}(\beta_1 \mu \bar{c}) + \left(H_c - \frac{\gamma_1}{\bar{c}}\right) J_{\gamma_1}(\beta_1 \mu \bar{c}),\\
&a_{12} = \beta_1 \mu Y_{\gamma_1+1}(\beta_1 \mu \bar{c}) + \left(H_c - \frac{\gamma_1}{\bar{c}}\right) Y_{\gamma_1}(\beta_1 \mu \bar{c}),\\
&a_{2N+2,2N+1} = (H_b + \gamma_{N+1}) J_{\gamma_{N+1}}(\beta_{N+1}\mu) - \beta_{N+1}\mu J_{\gamma_{N+1}+1}(\beta_{N+1}\mu),\\
&a_{2N+2,2N+2} = (H_b + \gamma_{N+1}) Y_{\gamma_{N+1}}(\beta_{N+1}\mu) - \beta_{N+1}\mu Y_{\gamma_{N+1}+1}(\beta_{N+1}\mu),\\
&a_{2i,2i-1} = J_{\gamma_i}(\beta_i \mu R_i), \quad a_{2i,2i} = Y_{\gamma_i}(\beta_i \mu R_i),\\
&a_{2i,2i+1} = -J_{\gamma_{i+1}}(\beta_{i+1}\mu R_i), \quad a_{2i,2i+2} = -Y_{\gamma_{i+1}}(\beta_{i+1}\mu R_i),\\
&a_{2i+1,2i-1} = \bar{\lambda}_{ri}\left\{\frac{\gamma_i}{R_i} J_{\gamma_i}(\beta_i \mu R_i) - \beta_i \mu J_{\gamma_i+1}(\beta_i \mu R_i)\right\},\\
&a_{2i+1,2i} = \bar{\lambda}_{ri}\left\{\frac{\gamma_i}{R_i} Y_{\gamma_i}(\beta_i \mu R_i) - \beta_i \mu Y_{\gamma_i+1}(\beta_i \mu R_i)\right\},\\
&a_{2i+1,2i+1} = -\bar{\lambda}_{r,i+1}\left\{\frac{\gamma_{i+1}}{R_i} J_{\gamma_{i+1}}(\beta_{i+1}\mu R_i) - \beta_{i+1}\mu J_{\gamma_{i+1}+1}(\beta_{i+1}\mu R_i)\right\},\\
&a_{2i+1,2i+2} = -\bar{\lambda}_{r,i+1}\left\{\frac{\gamma_{i+1}}{R_i} Y_{\gamma_{i+1}}(\beta_{i+1}\mu R_i) - \beta_{i+1}\mu Y_{\gamma_{i+1}+1}(\beta_{i+1}\mu R_i)\right\}; \quad i = 1\text{–}N
\end{aligned}
\tag{13}
$$

$$
c_{2N+2} = H_b \overline{T}_b \hat{f}_b(q) \tag{14}
$$

On the other hand, the element e_{kl} of the coefficient matrix $[e_{kl}]$ is omitted here for the sake of brevity. In Eq. (13), q represents the parameter of finite sine transformation with respect to the variable θ and a symbol $(\hat{\ })$ represents the image function. In Eqs. (11) and (12), $\Delta'(\mu_j)$, q_k, β_i and γ_i are

$$
\Delta'(\mu_j) = \left.\frac{\mathrm{d}\Delta}{\mathrm{d}\mu}\right|_{\mu=\mu_j}, \quad q_k = \frac{k\pi}{\theta_0}, \quad \beta_i = \frac{1}{\sqrt{\bar{\kappa}_{ri}}}, \quad \gamma_i = \sqrt{\frac{\bar{\kappa}_{\theta i}}{\bar{\kappa}_{ri}}} q_k \tag{15}
$$

and μ_j represent the jth positive roots of the following transcendental equation

$$
\Delta(\mu) = 0 \tag{16}
$$

2.2. Piezothermoelastic problem

We develop the analysis for transient piezothermoelasticity of a simply supported cylindrical composite panel composed of angle-ply laminae and piezoelectric material as a generalized plane deformation problem.

In the case of the piezoelectric layer of crystal class mm2, the stress–strain relations are expressed in dimensionless form as follows:

$$\begin{Bmatrix} \bar{\sigma}_{rri} \\ \bar{\sigma}_{\theta\theta i} \\ \bar{\sigma}_{zzi} \\ \bar{\sigma}_{\theta zi} \\ \bar{\sigma}_{rzi} \\ \bar{\sigma}_{r\theta i} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{11i} & \overline{C}_{12i} & \overline{C}_{13i} & 0 & 0 & 0 \\ \overline{C}_{12i} & \overline{C}_{22i} & \overline{C}_{23i} & 0 & 0 & 0 \\ \overline{C}_{13i} & \overline{C}_{23i} & \overline{C}_{33i} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{C}_{44i} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{C}_{55i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{C}_{66i} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{rri} \\ \bar{\varepsilon}_{\theta\theta i} \\ \bar{\varepsilon}_{zzi} \\ \bar{\gamma}_{\theta zi} \\ \bar{\gamma}_{rzi} \\ \bar{\gamma}_{r\theta i} \end{Bmatrix} - \begin{Bmatrix} \bar{\beta}_{ri}\overline{T}_i \\ \bar{\beta}_{\theta i}\overline{T}_i \\ \bar{\beta}_{zi}\overline{T}_i \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$- \begin{Bmatrix} \bar{e}_1 & 0 & 0 \\ \bar{e}_2 & 0 & 0 \\ \bar{e}_3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{e}_5 \\ 0 & \bar{e}_6 & 0 \end{Bmatrix} \begin{Bmatrix} \overline{E}_r \\ \overline{E}_\theta \\ \overline{E}_z \end{Bmatrix}; \quad i = 1 \tag{17}$$

where

$$\bar{\beta}_{ri} = \overline{C}_{11i}\bar{\alpha}_{ri} + \overline{C}_{12i}\bar{\alpha}_{\theta i} + \overline{C}_{13i}\bar{\alpha}_{zi}, \quad \bar{\beta}_{\theta i} = \overline{C}_{12i}\bar{\alpha}_{ri} + \overline{C}_{22i}\bar{\alpha}_{\theta i} + \overline{C}_{23i}\bar{\alpha}_{zi},$$
$$\bar{\beta}_{zi} = \overline{C}_{13i}\bar{\alpha}_{ri} + \overline{C}_{23i}\bar{\alpha}_{\theta i} + \overline{C}_{33i}\bar{\alpha}_{zi}; \qquad i = 1 \tag{18}$$

In the case of the angle-ply laminate ($i = 2$–$(N+1)$), the stress–strain relations for the global coordinate system (r, θ, z) are expressed in dimensionless form as follows:

$$\begin{Bmatrix} \bar{\sigma}_{zzi} \\ \bar{\sigma}_{\theta\theta i} \\ \bar{\sigma}_{rri} \\ \bar{\sigma}_{r\theta i} \\ \bar{\sigma}_{rzi} \\ \bar{\sigma}_{\theta zi} \end{Bmatrix} = \begin{bmatrix} \overline{Q}^*_{11i} & \overline{Q}^*_{12i} & \overline{Q}^*_{13i} & 0 & 0 & \overline{Q}^*_{16i} \\ \overline{Q}^*_{12i} & \overline{Q}^*_{22i} & \overline{Q}^*_{23i} & 0 & 0 & \overline{Q}^*_{26i} \\ \overline{Q}^*_{13i} & \overline{Q}^*_{23i} & \overline{Q}^*_{33i} & 0 & 0 & \overline{Q}^*_{36i} \\ 0 & 0 & 0 & \overline{Q}^*_{44i} & \overline{Q}^*_{45i} & 0 \\ 0 & 0 & 0 & \overline{Q}^*_{45i} & \overline{Q}^*_{55i} & 0 \\ \overline{Q}^*_{16i} & \overline{Q}^*_{26i} & \overline{Q}^*_{36i} & 0 & 0 & \overline{Q}^*_{66i} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{zzi} \\ \bar{\varepsilon}_{\theta\theta i} \\ \bar{\varepsilon}_{rri} \\ \bar{\gamma}_{r\theta i} \\ \bar{\gamma}_{rzi} \\ \bar{\gamma}_{\theta zi} \end{Bmatrix} - \begin{Bmatrix} \bar{\beta}_{zi}\overline{T}_i \\ \bar{\beta}_{\theta i}\overline{T}_i \\ \bar{\beta}_{ri}\overline{T}_i \\ 0 \\ 0 \\ \bar{\beta}_{\theta zi}\overline{T}_i \end{Bmatrix}; \quad i = 2\text{–}(N+1) \tag{19}$$

where

$$\bar{\beta}_{zi} = \overline{Q}^*_{11i}\bar{\alpha}_{zi} + \overline{Q}^*_{12i}\bar{\alpha}_{\theta i} + \overline{Q}^*_{13i}\bar{\alpha}_{ri} + \overline{Q}^*_{16i}\bar{\alpha}_{\theta zi}, \quad \bar{\beta}_{\theta i} = \overline{Q}^*_{12i}\bar{\alpha}_{zi} + \overline{Q}^*_{22i}\bar{\alpha}_{\theta i} + \overline{Q}^*_{23i}\bar{\alpha}_{ri} + \overline{Q}^*_{26i}\bar{\alpha}_{\theta zi},$$
$$\bar{\beta}_{ri} = \overline{Q}^*_{13i}\bar{\alpha}_{zi} + \overline{Q}^*_{23i}\bar{\alpha}_{\theta i} + \overline{Q}^*_{33i}\bar{\alpha}_{ri} + \overline{Q}^*_{36i}\bar{\alpha}_{\theta zi}, \quad \bar{\beta}_{\theta zi} = \overline{Q}^*_{16i}\bar{\alpha}_{zi} + \overline{Q}^*_{26i}\bar{\alpha}_{\theta i} + \overline{Q}^*_{36i}\bar{\alpha}_{ri} + \overline{Q}^*_{66i}\bar{\alpha}_{\theta zi}; \quad i = 2\text{–}(N+1) \tag{20}$$

The constitutive equations for the electric field in dimensionless form are given as

$$\overline{D}_r = \bar{e}_1\bar{\varepsilon}_{rr1} + \bar{e}_2\bar{\varepsilon}_{\theta\theta 1} + \bar{e}_3\bar{\varepsilon}_{zz1} + \bar{\eta}_1\overline{E}_r + \bar{p}_1\overline{T}_1, \quad \overline{D}_\theta = \bar{e}_6\bar{\gamma}_{r\theta 1} + \bar{\eta}_2\overline{E}_\theta, \quad \overline{D}_z = \bar{e}_5\bar{\gamma}_{rz1} + \bar{\eta}_3\overline{E}_z \tag{21}$$

We assume the displacement components for the global coordinate system as the state of a generalized plane deformation in the following forms:

$$\bar{u}_{ri} = \bar{u}_{ri}(\rho, \theta), \quad \bar{u}_{\theta i} = \bar{u}_{\theta i}(\rho, \theta), \quad \bar{u}_{zi} = \bar{u}_{zi}(\rho, \theta) \tag{22}$$

The relations between the electric field intensities and the electric potential ϕ in dimensionless form are defined by

$$\overline{E}_r = -\bar{\phi}_{,\rho}, \quad \overline{E}_\theta = -\rho^{-1}\bar{\phi}_{,\theta}, \quad \overline{E}_z = -\bar{\phi}_{,\bar{z}} = 0 \tag{23}$$

where a comma denotes partial differentiation with respect to the variable that follows. If the free charge is absent, the equation of electrostatics is expressed in dimensionless form as follows:

$$\overline{D}_{r,\rho} + \rho^{-1}(\overline{D}_r + \overline{D}_{\theta,\theta}) = 0 \tag{24}$$

The displacement–strain relations for the ith layer are expressed in dimensionless form as follows:

$$\bar{\varepsilon}_{rri} = \bar{u}_{ri,\rho}, \quad \bar{\varepsilon}_{\theta\theta i} = \rho^{-1}(\bar{u}_{\theta i,\theta} + \bar{u}_{ri}), \quad \bar{\varepsilon}_{zzi} = 0, \quad \bar{\gamma}_{\theta zi} = \rho^{-1}\bar{u}_{zi,\theta}, \quad \bar{\gamma}_{rzi} = \bar{u}_{zi,\rho},$$
$$\bar{\gamma}_{r\theta i} = \rho^{-1}(\bar{u}_{ri,\theta} - \bar{u}_{\theta i}) + \bar{u}_{\theta i,\rho}; \qquad i = 1\text{–}(N+1) \tag{25}$$

The equilibrium equations for the ith layer are expressed in dimensionless form as follows:

$$\bar{\sigma}_{rri,\rho} + \rho^{-1}\bar{\sigma}_{r\theta i,\theta} + \rho^{-1}(\bar{\sigma}_{rri} - \bar{\sigma}_{\theta\theta i}) = 0, \quad \bar{\sigma}_{r\theta i,\rho} + \rho^{-1}\bar{\sigma}_{\theta\theta i,\theta} + 2\rho^{-1}\bar{\sigma}_{r\theta i} = 0,$$
$$\bar{\sigma}_{rzi,\rho} + \rho^{-1}(\bar{\sigma}_{\theta zi,\theta} + \bar{\sigma}_{rzi}) = 0; \quad i = 1\text{–}(N+1) \tag{26}$$

In expressions (17)–(26), the following dimensionless values have been introduced:

$$\bar{\sigma}_{kli} = \frac{\sigma_{kli}}{\alpha_0 Y_0 T_0}, \quad (\bar{\varepsilon}_{kli}, \bar{\gamma}_{kli}) = \frac{(\varepsilon_{kli}, \gamma_{kli})}{\alpha_0 T_0}, \quad (\bar{u}_{ri}, \bar{u}_{\theta i}, \bar{u}_{zi}) = \frac{(u_{ri}, u_{\theta i}, u_{zi})}{\alpha_0 T_0 b}, \quad (\bar{\alpha}_{ki}, \bar{\alpha}_{\theta zi}) = \frac{(\alpha_{ki}, \alpha_{\theta zi})}{\alpha_0},$$
$$(\overline{C}_{kli}, \overline{Q}^*_{kli}) = \frac{(C_{kli}, Q^*_{kli})}{Y_0}, \quad \overline{E}_k = \frac{E_k|d_1|}{\alpha_0 T_0}, \quad \overline{D}_k = \frac{D_k}{\alpha_0 Y_0 T_0 |d_1|}, \quad \bar{\phi} = \frac{\phi|d_1|}{\alpha_0 T_0 b}, \quad \bar{e}_k = \frac{e_k}{Y_0|d_1|},$$
$$\bar{\eta}_k = \frac{\eta_k}{Y_0|d_1|^2}, \quad \bar{p}_1 = \frac{p_1}{\alpha_0 Y_0 |d_1|} \tag{27}$$

where σ_{kli} are the stress components, ε_{kli} are the normal strain components, γ_{kli} are the shearing strain, $(u_{ri}, u_{\theta i}, u_{zi})$ are the displacement components, α_{ki} and $\alpha_{\theta zi}$ are the coefficients of linear thermal expansion, C_{kli} are the elastic stiffness constants, Q^*_{kli} are the transformed elastic stiffness constants, D_k are the electric displacement components, e_k are the piezoelectric coefficients, η_k are the dielectric constants, p_1 is the pyroelectric constant, d_1 is the piezoelectric modulus and α_0 and Y_0 are the typical values of the coefficient of linear thermal expansion and Young's modulus of elasticity, respectively.

In the case of the piezoelectric layer, substituting Eqs. (23) and (25) into Eqs. (17) and (21), and later into Eqs. (24) and (26), the governing equations of the displacement components and the electric potential in dimensionless form are written as

$$\overline{C}_{11i}(\bar{u}_{ri,\rho\rho} + \rho^{-1}\bar{u}_{ri,\rho}) - \rho^{-2}(\overline{C}_{22i}\bar{u}_{ri} - \overline{C}_{66i}\bar{u}_{ri,\theta\theta}) + (\overline{C}_{12i} + \overline{C}_{66i})\rho^{-1}\bar{u}_{\theta i,\rho\theta} - (\overline{C}_{22i} + \overline{C}_{66i})\rho^{-2}\bar{u}_{\theta i,\theta}$$
$$+ \bar{e}_1\bar{\phi}_{,\rho\rho} + \bar{e}_6\rho^{-2}\bar{\phi}_{,\theta\theta} + (\bar{e}_1 - \bar{e}_2)\rho^{-1}\bar{\phi}_{,\rho} = \bar{\beta}_{ri}\overline{T}_{i,\rho} + \rho^{-1}(\bar{\beta}_{ri} - \bar{\beta}_{\theta i})\overline{T}_i; \quad i = 1 \tag{28}$$

$$(\overline{C}_{66i} + \overline{C}_{12i})\rho^{-1}\bar{u}_{ri,\rho\theta} + (\overline{C}_{66i} + \overline{C}_{22i})\rho^{-2}\bar{u}_{ri,\theta} + \overline{C}_{66i}(\rho^{-1}\bar{u}_{\theta i,\rho} - \rho^{-2}\bar{u}_{\theta i} + \bar{u}_{\theta i,\rho\rho}) + \overline{C}_{22i}\rho^{-2}\bar{u}_{\theta i,\theta\theta}$$
$$+ \rho^{-2}\bar{e}_6\bar{\phi}_{,\theta} + (\bar{e}_6 + \bar{e}_2)\rho^{-1}\bar{\phi}_{,\rho\theta} = \rho^{-1}\bar{\beta}_{\theta i}\overline{T}_{i,\theta}; \quad i = 1 \tag{29}$$

$$\overline{C}_{55i}(\bar{u}_{zi,\rho\rho} + \rho^{-1}\bar{u}_{zi,\rho}) + \overline{C}_{44i}\rho^{-2}\bar{u}_{zi,\theta\theta} = 0; \quad i = 1 \tag{30}$$

$$\bar{e}_1\bar{u}_{ri,\rho\rho} + (\bar{e}_1 + \bar{e}_2)\rho^{-1}\bar{u}_{ri,\rho} + \bar{e}_6\rho^{-2}\bar{u}_{ri,\theta\theta} + (\bar{e}_2 + \bar{e}_6)\rho^{-1}\bar{u}_{\theta i,\rho\theta} - \bar{e}_6\rho^{-2}\bar{u}_{\theta i,\theta} - \bar{\eta}_1(\bar{\phi}_{,\rho\rho} + \bar{\rho}^{-1}\bar{\phi}_{,\rho})$$
$$- \bar{\eta}_2\bar{\rho}^{-2}\bar{\phi}_{,\theta\theta} = -\bar{p}_1(\overline{T}_{i,\rho} + \rho^{-1}\overline{T}_i); \quad i = 1 \tag{31}$$

In the case of the angle-ply laminate ($i = 2\text{–}(N+1)$), substituting Eq. (25) into Eq. (19), and later into Eq. (26), the governing equations of the displacement components in dimensionless form are written as

$$\overline{Q}^*_{33i}(\bar{u}_{ri,\rho\rho} + \rho^{-1}\bar{u}_{ri,\rho}) - \rho^{-2}(\overline{Q}^*_{22i}\bar{u}_{ri} - \overline{Q}^*_{44i}\bar{u}_{ri,\theta\theta}) + (\overline{Q}^*_{23i} + \overline{Q}^*_{44i})\rho^{-1}\bar{u}_{\theta i,\rho\theta} - (\overline{Q}^*_{22i} + \overline{Q}^*_{44i})\rho^{-2}\bar{u}_{\theta i,\theta}$$
$$+ (\overline{Q}^*_{36i} + \overline{Q}^*_{45i})\rho^{-1}\bar{u}_{zi,\rho\theta} - \overline{Q}^*_{26i}\rho^{-2}\bar{u}_{zi,\theta} = \bar{\beta}_{ri}\overline{T}_{i,\rho} + (\bar{\beta}_{ri} - \bar{\beta}_{\theta i})\rho^{-1}\overline{T}_i; \quad i = 2\text{–}(N+1) \tag{32}$$

$$(\overline{Q}^*_{44i} + \overline{Q}^*_{23i})\rho^{-1}\bar{u}_{ri,\rho\theta} + (\overline{Q}^*_{44i} + \overline{Q}^*_{22i})\rho^{-2}\bar{u}_{ri,\theta} + \overline{Q}^*_{44i}(\rho^{-1}\bar{u}_{\theta i,\rho} - \rho^{-2}\bar{u}_{\theta i} + \bar{u}_{\theta i,\rho\rho}) + \overline{Q}^*_{22i}\rho^{-2}\bar{u}_{\theta i,\theta\theta}$$
$$+ \overline{Q}^*_{45i}(\bar{u}_{zi,\rho\rho} + 2\rho^{-1}\bar{u}_{zi,\rho}) + \overline{Q}^*_{26i}\rho^{-2}\bar{u}_{zi,\theta\theta} = \rho^{-1}\bar{\beta}_{\theta i}\overline{T}_{i,\theta}; \quad i = 2\text{–}(N+1) \tag{33}$$

$$(\overline{Q}^*_{36i}+\overline{Q}^*_{45i})\rho^{-1}\bar{u}_{ri,\rho\theta}+\overline{Q}^*_{26i}\rho^{-2}\bar{u}_{ri,\theta}+\overline{Q}^*_{45i}\bar{u}_{\theta i,\rho\rho}+\overline{Q}^*_{26i}\rho^{-2}\bar{u}_{\theta i,\theta\theta}+\overline{Q}^*_{55i}(\bar{u}_{zi,\rho\rho}+\rho^{-1}\bar{u}_{zi,\rho})$$
$$+\overline{Q}^*_{66i}\rho^{-2}\bar{u}_{zi,\theta\theta}=\bar{\beta}_{\theta zi}\rho^{-1}\overline{T}_{i,\theta};\quad i=2\text{–}(N+1) \tag{34}$$

If the inner and outer surfaces of the combined panel are traction free, and the interfaces of the each layer are perfectly bonded, then the boundary conditions of inner and outer surfaces and the conditions of continuity at the interfaces can be represented as follows:

$$\begin{aligned}
&\rho=\bar{c};\quad \bar{\sigma}_{rr1}=0,\quad \bar{\sigma}_{r\theta 1}=0,\quad \bar{\sigma}_{rz1}=0\\
&\rho=1;\quad \bar{\sigma}_{rr,N+1}=0,\quad \bar{\sigma}_{r\theta,N+1}=0,\quad \bar{\sigma}_{rz,N+1}=0\\
&\rho=R_i;\quad \bar{\sigma}_{rri}=\bar{\sigma}_{rr,i+1},\quad \bar{\sigma}_{r\theta i}=\bar{\sigma}_{r\theta,i+1},\quad \bar{\sigma}_{rzi}=\bar{\sigma}_{rz,i+1},\\
&\qquad\quad \bar{u}_{ri}=\bar{u}_{r,i+1},\quad \bar{u}_{\theta i}=\bar{u}_{\theta,i+1},\quad \bar{u}_{zi}=\bar{u}_{z,i+1};\quad i=1\text{–}N
\end{aligned} \tag{35}$$

The boundary conditions in the radial direction for the electric field are expresses by

$$\rho=\bar{c};\quad \overline{D}_r=0,\quad \rho=\bar{a};\quad \bar{\phi}=0 \tag{36}$$

We now consider the case of a simply supported panel and assume that the edges of the piezoelectric layer are electrically grounded. The boundary conditions are given as follows:

$$\theta=0,\ \theta_0;\quad \bar{\sigma}_{\theta\theta i}=0,\quad \bar{\sigma}_{\theta zi}=0,\quad \bar{u}_{ri}=0,\quad \bar{\phi}=0 \tag{37}$$

We assume the solutions of the displacement components and electric potential in order to satisfy Eq. (37) in the following form:

$$\begin{aligned}
\bar{u}_{ri}&=\sum_{k=1}^{\infty}\{U_{rcik}(\rho)+U_{rpik}(\rho)\}\sin q_k\theta,\\
\bar{u}_{\theta i}&=\sum_{k=1}^{\infty}\{U_{\theta cik}(\rho)+U_{\theta pik}(\rho)\}\cos q_k\theta,\\
\bar{u}_{zi}&=\sum_{k=1}^{\infty}\{U_{zcik}(\rho)+U_{zpik}(\rho)\}\cos q_k\theta;\quad i=1\text{–}(N+1),\\
\bar{\phi}&=\sum_{k=1}^{\infty}\{\Phi_{ck}(\rho)+\Phi_{pk}(\rho)\}\sin q_k\theta
\end{aligned} \tag{38}$$

In expression (38), the first term on the right side gives the homogeneous solution and the second term of right side gives the particular solution. However, since Eq. (30) has not the term of the temperature, the particular solution $U_{zpik}(\rho)$ of the piezoelectric layer does not exist. We now consider the homogeneous solution and introduce the following equation:

$$\rho=\exp(s) \tag{39}$$

Substituting the first term on the right side of Eq. (38) into the homogeneous expression of Eqs. (28)–(31), and later changing a variable with the use of Eq. (38), we have for the piezoelectric layer

$$[\overline{C}_{11i}\overline{D}^2-(\overline{C}_{22i}+\overline{C}_{66i}q_k^2)]U_{rcik}-[(\overline{C}_{12i}+\overline{C}_{66i})\overline{D}-(\overline{C}_{22i}+\overline{C}_{66i})]q_kU_{\theta cik}$$
$$+(\bar{e}_1\overline{D}^2-\bar{e}_2\overline{D}-\bar{e}_6q_k^2)\Phi_{ck}=0;\quad i=1 \tag{40}$$

$$[(\overline{C}_{12i}+\overline{C}_{66i})\overline{D}+\overline{C}_{22i}+\overline{C}_{66i}]q_kU_{rcik}+[\overline{C}_{66i}\overline{D}^2-(\overline{C}_{66i}+\overline{C}_{22i}q_k^2)]U_{\theta cik}$$
$$+[(\bar{e}_6+\bar{e}_2)\overline{D}+\bar{e}_6]q_k\Phi_{ck}=0;\quad i=1 \tag{41}$$

$$(\overline{C}_{55i}\overline{D}^2 - \overline{C}_{44i}q_k^2)U_{zcik} = 0; \quad i = 1 \tag{42}$$

$$(\bar{e}_1\overline{D}^2 + \bar{e}_2\overline{D} - \bar{e}_6 q_k^2)U_{rcik} - [(\bar{e}_2 + \bar{e}_6)\overline{D} - \bar{e}_6]q_k U_{\theta cik} - (\bar{\eta}_1\overline{D}^2 - \bar{\eta}_2 q_k^2)\Phi_{ck} = 0; \quad i = 1 \tag{43}$$

where

$$\overline{D} = \frac{\mathrm{d}}{\mathrm{d}s} \tag{44}$$

We show U_{rcik}, $U_{\theta cik}$, U_{zcik} and Φ_{ck} as follows:

$$(U_{rcik}, U_{\theta cik}, U_{zcik}, \Phi_{ck}) = (U^0_{rcik}, U^0_{\theta cik}, U^0_{zcik}, \Phi^0_{ck})\exp(\lambda_i s); \quad i = 1 \tag{45}$$

Substituting Eq. (45) into Eqs. (40), (41) and (43), the condition that a non-trivial solution of $(U^0_{rcik}, U^0_{\theta cik}, \Phi^0_{ck})$ exists leads to the following equation:

$$p_i^3 + d^{(i)}p_i + f^{(i)} = 0 \tag{46}$$

where

$$p_i = \lambda_i^2 - \frac{B^{(i)}}{3A^{(i)}}, \quad d^{(i)} = -\left[\frac{3A^{(i)}C^{(i)} + (B^{(i)})^2}{3(A^{(i)})^2}\right],$$
$$f^{(i)} = -\left[\frac{2(B^{(i)})^3 + 9A^{(i)}B^{(i)}C^{(i)} + 27D^{(i)}(A^{(i)})^2}{27(A^{(i)})^3}\right] \tag{47}$$

$$\begin{aligned}
A^{(i)} &= \overline{C}_{66i}(\bar{e}_1^2 + \bar{\eta}_1\overline{C}_{11i}),\\
B^{(i)} &= q_k^2[\bar{\eta}_1(\overline{C}_{11i}\overline{C}_{22i} - \overline{C}_{12i}^2 - 2\overline{C}_{12i}\overline{C}_{66i}) + \bar{\eta}_2\overline{C}_{11i}\overline{C}_{66i}\\
&\quad + \overline{C}_{11i}(\bar{e}_2 + \bar{e}_6)^2 - 2\bar{e}_1\bar{e}_2(\overline{C}_{12i} + \overline{C}_{66i}) - 2\overline{C}_{12i}\bar{e}_1\bar{e}_6 + \bar{e}_1^2\overline{C}_{22i}]\\
&\quad + \overline{C}_{66i}[(\overline{C}_{11i} + \overline{C}_{22i})\bar{\eta}_1 + \bar{e}_1^2 + \bar{e}_2^2],\\
C^{(i)} &= -\overline{C}_{66i}(\overline{C}_{22i}\bar{\eta}_1 + \bar{e}_2^2)(q_k^2 - 1)^2 + 2q_k^2(q_k^2 - 1)\bar{e}_6(\overline{C}_{12i}\bar{e}_2 - \overline{C}_{22i}\bar{e}_1)\\
&\quad - q_k^2(\overline{C}_{11i} + \overline{C}_{22i})(\bar{e}_6^2 + \bar{\eta}_2\overline{C}_{66i}) + q_k^4[\bar{\eta}_2(\overline{C}_{12i}^2 + 2\overline{C}_{12i}\overline{C}_{66i} - \overline{C}_{11i}\overline{C}_{22i}) + 2\overline{C}_{12i}\bar{e}_6^2],\\
D^{(i)} &= \overline{C}_{22i}q_k^2(\bar{e}_6^2 + \bar{\eta}_2\overline{C}_{66i})(q_k^2 - 1)^2
\end{aligned} \tag{48}$$

We now introduce the following expression:

$$H_i = \frac{(f^{(i)})^2}{4} + \frac{(d^{(i)})^3}{27} \tag{49}$$

From Eq. (46), there might be three distinct real roots, three real roots with at least two of them being equal or a real root in conjunction with one pair of conjugate complex roots depending H_i is negative, zero or positive, respectively. For instance, $U_{rcik}(\rho)$, $U_{\theta cik}(\rho)$ and $\Phi_{ck}(\rho)$ can be expressed as follows when $H_i < 0$:

$$U_{rcik}(\rho) = \sum_{J=1}^{3} U^J_{rcik}(\rho), \quad U_{\theta cik}(\rho) = \sum_{J=1}^{3} U^J_{\theta cik}(\rho), \quad \Phi_{ck}(\rho) = \sum_{J=1}^{3} \Phi^J_{ck}(\rho) \tag{50}$$

where

$$
\begin{aligned}
&U_{rcik}^{J}(\rho) = F_{rJ}^{(i)}\rho^{m_{Ji}} + S_{rJ}^{(i)}\rho^{-m_{Ji}},\\
&U_{\theta cik}^{J}(\rho) = L_{kiJ}(m_{Ji})F_{rJ}^{(i)}\rho^{m_{Ji}} + L_{kiJ}(-m_{Ji})S_{rJ}^{(i)}\rho^{-m_{Ji}},\\
&\Phi_{ck}^{J}(\rho) = R_{kiJ}(m_{Ji})F_{rJ}^{(i)}\rho^{m_{Ji}} + R_{kiJ}(-m_{Ji})S_{rJ}^{(i)}\rho^{-m_{Ji}},\\
&m_{Ji} = \sqrt{p_{iJ} + \frac{B^{(i)}}{3A^{(i)}}} \quad \text{if } p_{iJ} + \frac{B^{(i)}}{3A^{(i)}} > 0
\end{aligned}
\tag{51}
$$

$$
\begin{aligned}
&U_{rcik}^{J}(\rho) = F_{rJ}^{(i)}\cos(m_{Ji}\ln\rho) + S_{rJ}^{(i)}\sin(m_{Ji}\ln\rho),\\
&U_{\theta cik}^{J}(\rho) = \{\mathrm{Re}[L_{kiJ}(\mathrm{j}m_{Ji})]\cos(m_{Ji}\ln\rho) - \mathrm{Im}[L_{kiJ}(\mathrm{j}m_{Ji})]\sin(m_{Ji}\ln\rho)\}F_{rJ}^{(i)}\\
&\qquad + \{\mathrm{Im}[L_{kiJ}(\mathrm{j}m_{Ji})]\cos(m_{Ji}\ln\rho) + \mathrm{Re}[L_{kiJ}(\mathrm{j}m_{Ji})]\sin(m_{Ji}\ln\rho)\}S_{rJ}^{(i)},\\
&\Phi_{ck}^{J}(\rho) = \{\mathrm{Re}[R_{kiJ}(\mathrm{j}m_{Ji})]\cos(m_{Ji}\ln\rho) - \mathrm{Im}[R_{kiJ}(\mathrm{j}m_{Ji})]\sin(m_{Ji}\ln\rho)\}F_{rJ}^{(i)}\\
&\qquad + \{\mathrm{Im}[R_{kiJ}(\mathrm{j}m_{Ji})]\cos(m_{Ji}\ln\rho) + \mathrm{Re}[R_{kiJ}(\mathrm{j}m_{Ji})]\sin(m_{Ji}\ln\rho)\}S_{rJ}^{(i)},\\
&m_{Ji} = \sqrt{-\left(p_{iJ} + \frac{B^{(i)}}{3A^{(i)}}\right)} \quad \text{if } p_{iJ} + \frac{B^{(i)}}{3A^{(i)}} < 0
\end{aligned}
\tag{52}
$$

In Eqs. (51) and (52),

$$
\begin{aligned}
L_{kiJ}(x) &= \frac{q_k}{I_{kiJ}(x)}\{x^3[\bar{e}_1(\overline{C}_{12i} + \overline{C}_{66i}) - \overline{C}_{11i}(\bar{e}_2 + \bar{e}_6)]\\
&\quad + x^2[\bar{e}_1(\overline{C}_{22i} + \overline{C}_{66i}) - \bar{e}_2(\overline{C}_{12i} + \overline{C}_{66i}) - \overline{C}_{11i}\bar{e}_6]\\
&\quad + x[(\bar{e}_6\overline{C}_{22i} - \bar{e}_2\overline{C}_{66i}) + q_k^2(\bar{e}_2\overline{C}_{66i} - \bar{e}_6\overline{C}_{12i})] + \overline{C}_{22i}\bar{e}_6(1 - q_k^2)\},\\
R_{kiJ}(x) &= \frac{1}{I_{kiJ}(x)}[\overline{C}_{11i}\overline{C}_{66i}x^4 + x^2\{[(\overline{C}_{12i} + \overline{C}_{66i})^2\\
&\quad - \overline{C}_{11i}\overline{C}_{22i} - \overline{C}_{66i}^2]q_k^2 - \overline{C}_{66i}(\overline{C}_{11i} + \overline{C}_{22i})\} + \overline{C}_{22i}\overline{C}_{66i}(1 - q_k^2)^2],\\
I_{kiJ}(x) &= -\overline{C}_{66i}\bar{e}_1x^4 + \overline{C}_{66i}\bar{e}_2x^3 + x^2\{q_k^2[\overline{C}_{22i}\bar{e}_1 + \overline{C}_{66i}\bar{e}_6 - (\overline{C}_{12i} + \overline{C}_{66i})(\bar{e}_2 + \bar{e}_6)] + \overline{C}_{66i}\bar{e}_1\}\\
&\quad + x\{q_k^2[\bar{e}_6(\overline{C}_{22i} - \overline{C}_{12i}) + \bar{e}_2\overline{C}_{66i}] - \bar{e}_2\overline{C}_{66i}\} + \overline{C}_{22i}\bar{e}_6q_k^2(1 - q_k^2)
\end{aligned}
\tag{53}
$$

In Eq. (52), j, Re[ ] and Im[ ] are imaginary unit $\mathrm{j} = \sqrt{-1}$, real part and imaginary part, respectively. Furthermore, in Eqs. (51) and (52), $F_{rJ}^{(i)}$ and $S_{rJ}^{(i)}$ are unknown constants. The case of $H_i = 0$ or $H_i > 0$ is omitted here for the sake of brevity.

Substituting Eq. (45) into Eq. (42), the condition that non-trivial solutions of U_{zcik}^0 exist leads to the following equation:

$$
\lambda_i = \pm m_{4i}; \quad i = 1 \tag{54}
$$

where

$$
m_{4i} = \pm\sqrt{\frac{\overline{C}_{44i}}{\overline{C}_{55i}}}\,q_k; \quad i = 1 \tag{55}
$$

Then $U_{zc1k}(\rho)$ can be expressed as follows:

$$
U_{zc1k}(\rho) = F_z\rho^{m_{41}} + S_z\rho^{-m_{41}} \tag{56}
$$

In Eq. (56), F_z and S_z are unknown constants.

In the case of angle-ply laminate, $U_{rcik}(\rho)$, $U_{\theta cik}(\rho)$ and $U_{zcik}(\rho)$ are obtained in the same way as the case of the piezoelectric layer. Assuming that

$$(U_{rcik}, U_{\theta cik}, U_{zcik}) = (U^0_{rcik}, U^0_{\theta cik}, U^0_{zcik})\exp(\lambda_i s); \quad i = 2\text{–}(N+1) \tag{57}$$

we get the same equation of Eqs. (46) and (47). In this case, Eq. (48) exchanges to the next equation.

$$\begin{aligned}
A^{(i)} &= \overline{Q}^*_{33i}[\overline{Q}^*_{55i}\overline{Q}^*_{44i} - (\overline{Q}^*_{45i})^2],\\
B^{(i)} &= q_k^2\overline{Q}^*_{55i}[\overline{Q}^*_{33i}\overline{Q}^*_{22i} - (\overline{Q}^*_{23i})^2] + q_k^2\overline{Q}^*_{44i}[\overline{Q}^*_{33i}\overline{Q}^*_{66i} - (\overline{Q}^*_{36i})^2]\\
&\quad + (\overline{Q}^*_{33i} + \overline{Q}^*_{22i} - 2q_k^2\overline{Q}^*_{23i})[\overline{Q}^*_{55i}\overline{Q}^*_{44i} - (\overline{Q}^*_{45i})^2] - 2q_k^2\overline{Q}^*_{45i}(\overline{Q}^*_{33i}\overline{Q}^*_{26i} - \overline{Q}^*_{23i}\overline{Q}^*_{36i}),\\
C^{(i)} &= [q_k^4\overline{Q}^*_{36i} + 2q_k^2(q_k^2-1)\overline{Q}^*_{45i}](\overline{Q}^*_{22i}\overline{Q}^*_{36i} - \overline{Q}^*_{23i}\overline{Q}^*_{26i}) - q_k^4\overline{Q}^*_{33i}[\overline{Q}^*_{22i}\overline{Q}^*_{66i} - (\overline{Q}^*_{26i})^2]\\
&\quad + q_k^4(\overline{Q}^*_{23i} + 2\overline{Q}^*_{44i})(\overline{Q}^*_{23i}\overline{Q}^*_{66i} - \overline{Q}^*_{36i}\overline{Q}^*_{26i}) - \overline{Q}^*_{22i}(q_k^2-1)^2[\overline{Q}^*_{55i}\overline{Q}^*_{44i} - (\overline{Q}^*_{45i})^2]\\
&\quad + q_k^2\overline{Q}^*_{44i}[(\overline{Q}^*_{36i})^2 + (\overline{Q}^*_{26i})^2 - \overline{Q}^*_{33i}\overline{Q}^*_{66i} - \overline{Q}^*_{22i}\overline{Q}^*_{66i}],\\
D^{(i)} &= q_k^2(q_k^2-1)^2\overline{Q}^*_{44i}[\overline{Q}^*_{22i}\overline{Q}^*_{66i} - (\overline{Q}^*_{26i})^2]
\end{aligned} \tag{58}$$

Therefore $(U_{rcik}, U_{\theta cik}, U_{zcik})$ of angle-ply laminate are obtained as the function systems like $(U_{rcik}, U_{\theta cik}, \Phi_{ck})$ of the piezoelectric layer ($i = 1$). $(U_{rcik}, U_{\theta cik}, U_{zcik})$ of angle-ply laminate had been obtained in the previous paper (Ootao and Tanigawa, 2002), and their details are omitted here.

Next, in order to obtain the particular solution, we use the series expansions of the Bessel function as follows:

$$J_\gamma(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n+\gamma}}{n!\Gamma(\gamma+n+1)} \tag{59}$$

$$Y_\gamma(x) = \frac{1}{\sin\gamma\pi}[\cos\gamma\pi J_\gamma(x) - J_{-\gamma}(x)] \quad \text{if } \gamma \neq \text{integer} \tag{60}$$

Since the order γ_i of the Bessel function in Eq. (12) is not integer in general except ply angle $\phi_i = 0°$, Eq. (12) can be written as the following expression using Eqs. (59) and (60).

$$\overline{T}_{ik}(\rho, \tau) = \sum_{n=0}^{\infty}\{a_{ni}(\tau)\rho^{2n+\gamma_i} + b_{ni}(\tau)\rho^{2n-\gamma_i}\} \tag{61}$$

Expressions for the functions $a_{ni}(\tau)$ and $b_{ni}(\tau)$ in Eq. (61) are omitted here for the sake of brevity. We assume $U_{rpik}(\rho)$, $U_{\theta pik}(\rho)$, $U_{zpik}(\rho)$ and $\Phi_{pk}(\rho)$ as follows:

$$\begin{aligned}
U_{rpik}(\rho) &= \sum_{n=0}^{\infty}\left(f_{ani}\rho^{2n+\gamma_i+1} + f_{bni}\rho^{2n-\gamma_i+1}\right); \quad i = 1\text{–}(N+1),\\
U_{\theta pik}(\rho) &= \sum_{n=0}^{\infty}\left(g_{ani}\rho^{2n+\gamma_i+1} + g_{bni}\rho^{2n-\gamma_i+1}\right); \quad i = 1\text{–}(N+1),\\
U_{zpik}(\rho) &= \sum_{n=0}^{\infty}\left(h_{ani}\rho^{2n+\gamma_i+1} + h_{bni}\rho^{2n-\gamma_i+1}\right); \quad i = 2\text{–}(N+1),\\
\Phi_{pk}(\rho) &= \sum_{n=0}^{\infty}\left(i_{ani}\rho^{2n+\gamma_i+1} + i_{bni}\rho^{2n-\gamma_i+1}\right); \quad i = 1
\end{aligned} \tag{62}$$

Substituting Eqs. (11), (61), (62) and the second term of right side of Eq. (38) into Eqs. (28), (29), (31) or Eqs. (32)–(34), and later comparing the coefficients of functions with regard to ρ respectively, the constants f_{ani}, f_{bni}, g_{ani}, g_{bni}, h_{ani}, h_{bni}, i_{ani} and i_{bni} can be obtained.

Then, in the case of angle-ply laminate, the stress components can be evaluated by substituting the first three of Eq. (38) into the Eq. (25), and later Eq. (19). In the case of the piezoelectric layer, the stress components and the electric displacements can be evaluated by substituting Eq. (38) into the Eqs. (23) and (25), and later into Eqs. (17) and (21). The unknown constants in the homogeneous solutions such as Eqs. (51), (52) and (56) are determined so as to satisfy the boundary conditions (35) and (36).

3. Numerical results

We consider the piezoelectric layer composed of a cadmium selenide solid and the angle-ply laminate composed of alumina fiber reinforced aluminum composite. We assume that each layer of angle-ply laminated panel consists of the same orthotropic material, and consider a two-layered anti-symmetric angle-ply laminated panel with the fiber-orientation (60°/−60°) and the same thickness. We assume that the combined cylindrical panel is heated by surrounding media, the temperature of which is denoted by the symmetric function with respect to the center of the panel ($\theta = \theta_0/2$). Then, numerical calculative parameters of heat condition and shape are presented as follows:

$$H_b = H_c = 1.0, \quad \overline{T}_b = 1, \quad \theta_0 = 90^\circ, \quad \bar{a} = 0.7, 0.85, 0.95, \quad \bar{a} - \bar{c} = 0.05,$$
$$f_b(\theta) = (1 - \theta'^2/\theta_b^2)H(\theta_b - |\theta'|), \quad \theta_b = 15^\circ, \theta' = \theta - \theta_0/2 \tag{63}$$

where $H(x)$ is Heaviside's function. The material constants (Ootao and Tanigawa, 2000a) are taken as for alumina fiber reinforced aluminum composite,

$$\kappa_L = 41.1 \times 10^{-6}\ \mathrm{m^2/s}, \quad \kappa_T = 29.5 \times 10^{-6}\ \mathrm{m^2/s}, \quad \alpha_L = 7.6 \times 10^{-6}\ \mathrm{K^{-1}},$$
$$\alpha_T = 14.0 \times 10^{-6}\ \mathrm{K^{-1}}, \quad \lambda_L = 105\ \mathrm{W/mK}, \quad \lambda_T = 75\ \mathrm{W/mK}, \quad Y_L = 150\ \mathrm{GPa},$$
$$Y_T = 110\ \mathrm{GPa}, G_{LT} = 35\mathrm{GPa}, \quad G_{TT} = 41\ \mathrm{GPa}, \quad \nu_{LT} = 0.33, \quad \nu_{TT} = 0.33, \quad \nu_{TL} = 0.242 \tag{64}$$

and for cadmium selenide,

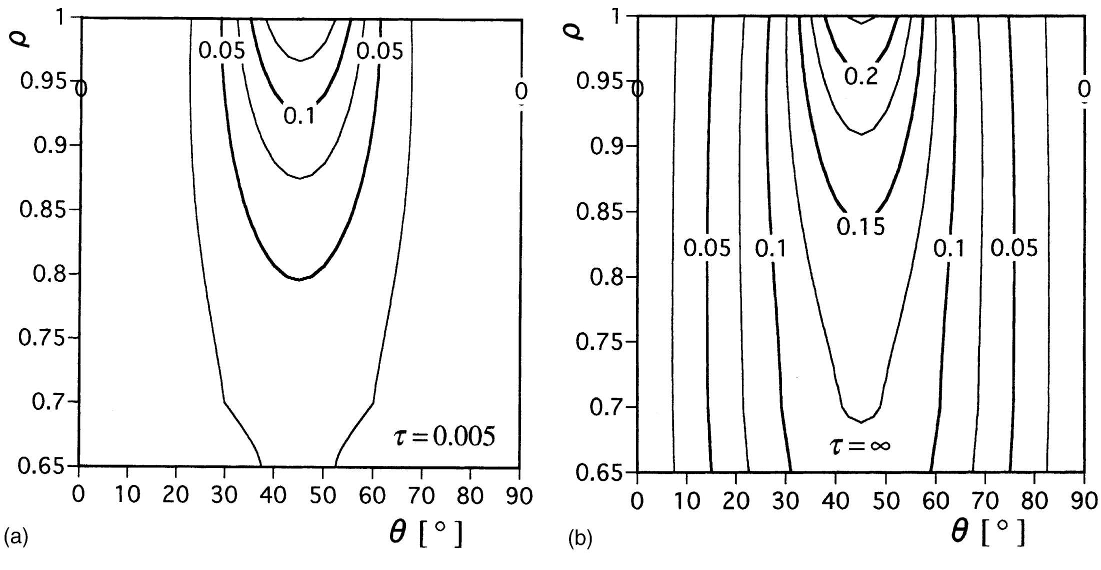

Fig. 1. Temperature change distribution in a (a) transient state ($\tau = 0.005$) and (b) steady state ($\tau = \infty$).

$$\begin{aligned}
&\alpha_\theta = \alpha_z = 4.396 \times 10^{-6}\ \mathrm{K}^{-1}, \quad \alpha_r = 2.458 \times 10^{-6}\ \mathrm{K}^{-1}, \quad C_{11} = 83.6\ \mathrm{GPa}, \quad C_{22} = C_{33} = 74.1\ \mathrm{GPa},\\
&C_{23} = 45.2\ \mathrm{GPa}, \quad C_{12} = C_{13} = 39.3\ \mathrm{GPa}, \quad C_{66} = 13.17\ \mathrm{GPa}, \quad e_1 = 0.347\ \mathrm{C/m^2},\\
&e_2 = e_3 = -0.16\ \mathrm{C/m^2}, \quad e_6 = -0.138\ \mathrm{C/m^2}, \quad \eta_1 = 9.03 \times 10^{-11}\ \mathrm{C^2/Nm^2},\\
&\eta_2 = 8.25 \times 10^{-11}\ \mathrm{C^2/Nm^2} \quad p_1 = -2.94 \times 10^{-6}\ \mathrm{C/m^2K},\\
&d_1 = -3.92 \times 10^{-12}\ \mathrm{C/N} \quad \lambda_\theta = 8.6\ \mathrm{W/mK}, \quad \lambda_r = 1.5\lambda_\theta
\end{aligned} \tag{65}$$

where G and v are the shear modulus of elasticity and Poisson's ratio, respectively. Since the coefficients of thermal conductivity for cadmium selenide could not be found in the literature, the following values are assumed:

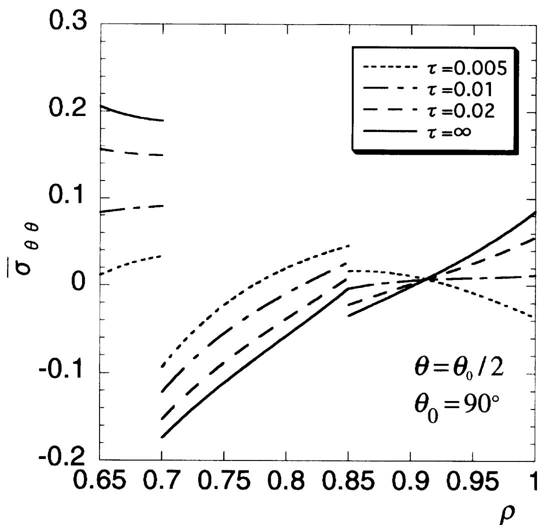

Fig. 2. Variation of the thermal stress $\bar{\sigma}_{\theta\theta}$ in the radial direction ($\theta = \theta_0/2$).

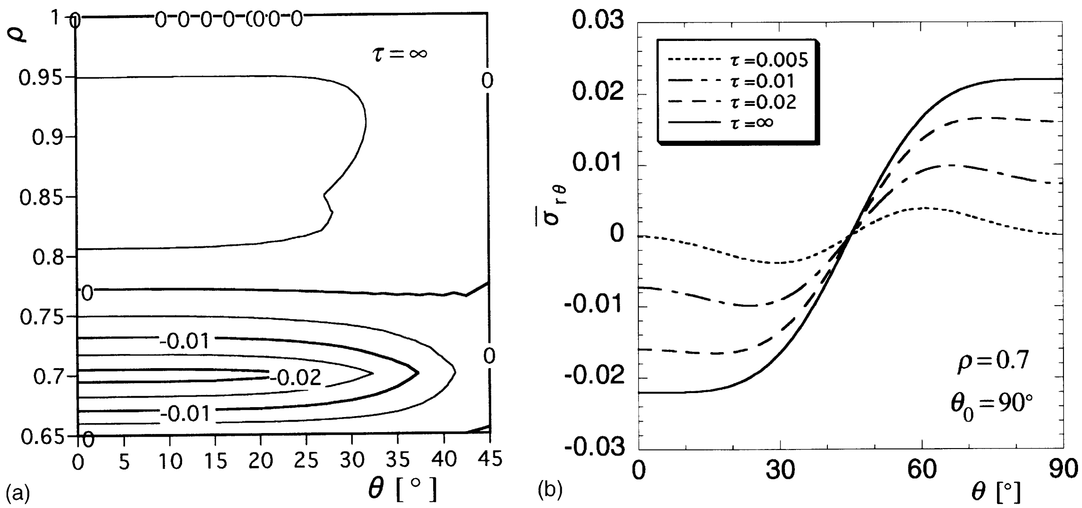

Fig. 3. Thermal stress $\bar{\sigma}_{r\theta}$: (a) distribution in a steady state ($\tau = \infty$) and (b) variation on the interface ($\rho = 0.7$).

$$\kappa_\theta = 3.28 \times 10^{-6}\ \mathrm{m^2/s}, \quad \kappa_r = 1.5\kappa_\theta \tag{66}$$

The typical values of material properties such as κ_0, λ_0, α_0 and Y_0, used to normalize the numerical data, are based on those of cadmium selenide as follows:

$$\kappa_0 = \kappa_\theta, \quad \lambda_0 = \lambda_\theta, \quad \alpha_0 = \alpha_\theta, \quad Y_0 = 42.8\ \mathrm{GPa} \tag{67}$$

Figs. 1–5 show the numerical results for $\bar{a} = 0.7$. Fig. 1 shows the results of temperature change. The distribution in a transient state ($\tau = 0.005$) is shown in Fig. 1(a) and the distribution in a steady state is shown in Fig. 1(b). As shown in Fig. 1, the temperature rise can clearly be seen in the heated region. Fig. 2 shows the variation of the normal stress $\bar{\sigma}_{\theta\theta}$ at the midpoint of the panel. From Fig. 2, discontinuities of stress occur on the interfaces. In order to valuate the phenomenon of delamination, it is necessary to focus

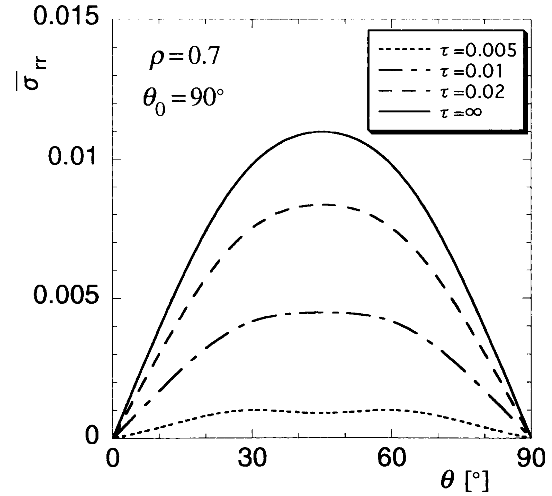

Fig. 4. Variation of the thermal stress $\bar{\sigma}_{rr}$ on the interface ($\rho = 0.7$).

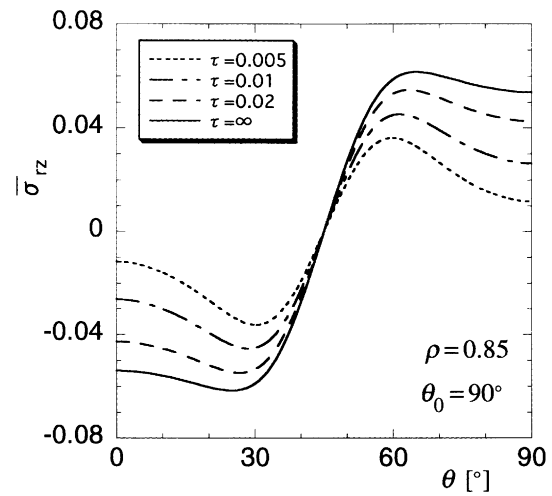

Fig. 5. Variation of the thermal stress $\bar{\sigma}_{rz}$ on the interface ($\rho = 0.85$).

attention on transeverse stresses. Then, Figs. 3–5 show the variations of the shearing stress $\bar{\sigma}_{r\theta}$, the normal stress $\bar{\sigma}_{rr}$ and the shearing stress $\bar{\sigma}_{rz}$, respectively. The distributions in a steady state are shown in Fig. 3(a), and the variation on the interface ($\rho = 0.7$) is shown in Fig. 3(b). Since the shearing stress $\bar{\sigma}_{r\theta}$ is anti-symmetric with respect to $\theta = 45°$ under the condition of Eq. (63), Fig. 3(a) shows a range of 0–45°. From Fig. 3(a), it can be seen that the stress of $\bar{\sigma}_{r\theta}$ shows the maximum value on the interface between the piezoelectric layer and the angle-ply laminae ($\rho = 0.7$). Fig. 4 shows the variation on the interface ($\rho = 0.7$), because the tensile stress of $\bar{\sigma}_{rr}$ shows the maximum value on the interface ($\rho = 0.7$). Fig. 5 shows the variation on the interface ($\rho = 0.85$), because the stress of $\bar{\sigma}_{rz}$ shows the maximum value on the interface between the second layer and the third layer ($\rho = 0.85$). As shown in Figs. 3(b), 4 and 5, it can be seen that

Table 1
Effect of thickness of angle-ply laminate for $\tau = 0.01$

$\tau = 0.01$	$\bar{a} = 0.7$, $\bar{c} = 0.65$	$\bar{a} = 0.85$, $\bar{c} = 0.8$	$\bar{a} = 0.95$, $\bar{c} = 0.9$
$\overline{T}(\rho = 1, \theta = 45°)$	0.1802	0.2202	0.3495
$\overline{T}(\rho = \bar{c}, \theta = 45°)$	0.06127	0.1414	0.2994
$\bar{u}_r(\rho = 1, \theta = 45°)$	0.1589	0.2788	1.544
$\bar{u}_r(\rho = \bar{c}, \theta = 45°)$	0.02838	0.1653	1.464
$\bar{u}_\theta(\rho = 1, \theta = 0°)$	−0.09606	−0.09781	0.3839
$\bar{u}_\theta(\rho = \bar{c}, \theta = 0°)$	−0.02733	−0.009072	0.6065
$\bar{u}_z(\rho = 1, \theta = 0°)$	−0.01007	−0.007609	−0.007982
$\bar{u}_z(\rho = \bar{c}, \theta = 0°)$	−0.005683	−0.006373	−0.007861
$\bar{\sigma}_{rr}(\rho = \bar{a}, \theta = 45°)$	0.004502	0.006424	0.005593
$\bar{\sigma}_{\theta\theta}(\rho = 1, \theta = 45°)$	0.01184	0.1469	0.4567
$\bar{\sigma}_{\theta\theta}(\rho = \bar{a}_+, \theta = 45°)$	−0.1215	−0.3181	−0.7617
$\bar{\sigma}_{\theta\theta}(\rho = \bar{c}, \theta = 45°)$	0.08357	0.1430	−0.09534
$\bar{\sigma}_{zz}(\rho = 1, \theta = 45°)$	−1.193	−1.412	−2.162
$\bar{\sigma}_{zz}(\rho = \bar{c}, \theta = 45°)$	−0.01864	−0.06751	−0.3383
$\bar{\sigma}_{r\theta}(\rho = \bar{a}, \theta = 30°)$	−0.09362	−0.01898	−0.01794
$\bar{\sigma}_{rz}(\rho = \bar{a} + (1 - \bar{a})/2, \theta = 30°)$	−0.04709	−0.04594	−0.03346
$\bar{\phi} \times 10^3(\rho = \bar{c}, \theta = 45°)$	0.1278	0.2765	0.4729

Table 2
Effect of thickness of angle-ply laminate for $\tau = \infty$

$\tau = \infty$	$\bar{a} = 0.7$, $\bar{c} = 0.65$	$\bar{a} = 0.85$, $\bar{c} = 0.8$	$\bar{a} = 0.95$, $\bar{c} = 0.9$
$\overline{T}(\rho = 1, \theta = 45°)$	0.2294	0.2946	0.4285
$\overline{T}(\rho = \bar{c}, \theta = 45°)$	0.1198	0.2257	0.3867
$\bar{u}_r(\rho = 1, \theta = 45°)$	0.1454	0.3171	1.964
$\bar{u}_r(\rho = \bar{c}, \theta = 45°)$	−0.05199	0.1534	1.865
$\bar{u}_\theta(\rho = 1, \theta = 0°)$	−0.1877	−0.2050	0.4640
$\bar{u}_\theta(\rho = \bar{c}, \theta = 0°)$	−0.1247	−0.09157	0.7592
$\bar{u}_z(\rho = 1, \theta = 0°)$	−0.01285	−0.01028	−0.01053
$\bar{u}_z(\rho = \bar{c}, \theta = 0°)$	−0.003950	−0.007483	−0.01024
$\bar{\sigma}_{rr}(\rho = \bar{a}, \theta = 45°)$	0.01105	0.01148	0.007281
$\bar{\sigma}_{\theta\theta}(\rho = 1, \theta = 45°)$	0.08559	0.2527	0.5731
$\bar{\sigma}_{\theta\theta}(\rho = \bar{a}_+, \theta = 45°)$	−0.1738	−0.4747	−0.9470
$\bar{\sigma}_{\theta\theta}(\rho = \bar{c}, \theta = 45°)$	0.2058	0.2388	−0.1144
$\bar{\sigma}_{zz}(\rho = 1, \theta = 45°)$	−1.494	−1.869	−2.646
$\bar{\sigma}_{zz}(\rho = \bar{c}, \theta = 45°)$	−0.01563	−0.1026	−0.4328
$\bar{\sigma}_{r\theta}(\rho = \bar{a}, \theta = 30°)$	−0.01686	−0.02476	−0.02005
$\bar{\sigma}_{rz}(\rho = \bar{a} + (1 - \bar{a})/2, \theta = 30°)$	−0.06099	−0.05578	−0.03721
$\bar{\phi} \times 10^3(\rho = \bar{c}, \theta = 45°)$	0.2583	0.4360	0.6032

the values of shearing stresses $\bar{\sigma}_{r\theta}$ and $\bar{\sigma}_{rz}$ and normal stress $\bar{\sigma}_{rr}$ rise as the time proceeds and have maximum value in the steady state. Figs. 3(b), 4 and 5 show that the values of shearing stress $\bar{\sigma}_{rz}$ are larger than those of normal stress $\bar{\sigma}_{rr}$ and shearing stress $\bar{\sigma}_{r\theta}$.

In order to examine the influence of thickness of angle-ply laminate, the numerical results for $\bar{a} = 0.7$, 0.85, 0.95 are shown in Tables 1 and 2 and Figs. 6 and 7. Tables 1 and 2 show the typical values of temperature change, displacement, stress, and electric potential for a transient state ($\tau = 0.01$) and ones for the steady state, respectively. In Tables 1 and 2, $\bar{\sigma}_{\theta\theta}$ at $\rho = \bar{a}_+$ shows the value of the second layer. From Tables 1 and 2, the values of temperature change increase when the radial ratio $\bar{a}$ increases, that is, when the thickness of angle-ply laminate decreases. Fig. 6 shows the variations of thermal displacement. The variation of the thermal displacement $\bar{u}_\theta$ on the cross-section $\theta = 0°$ is shown in Fig. 6(a) and the variation of

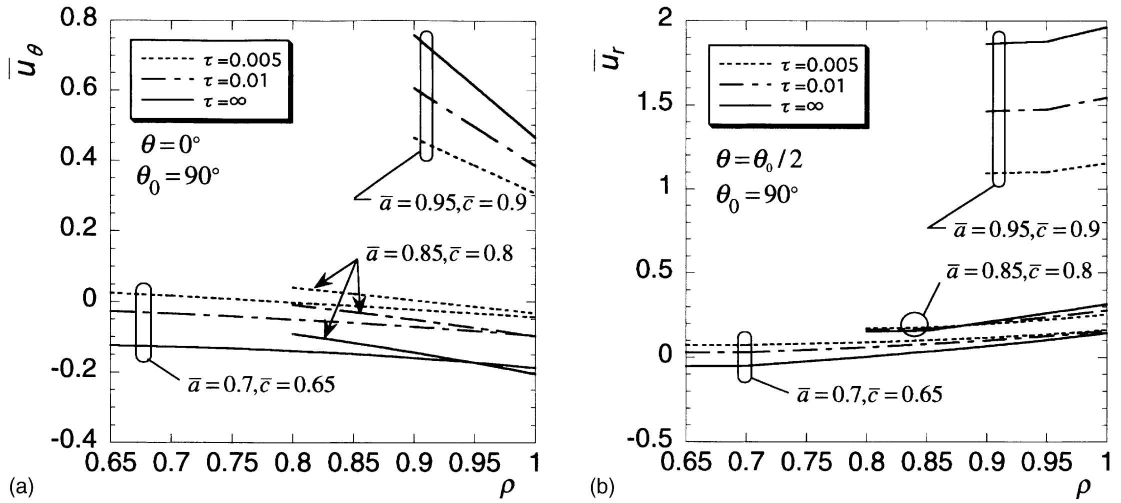

Fig. 6. Variation of the thermal displacement (a) $\bar{u}_\theta$ in the radial direction ($\theta = 0°$) and (b) $\bar{u}_r$ in the radial direction ($\theta = \theta_0/2$).

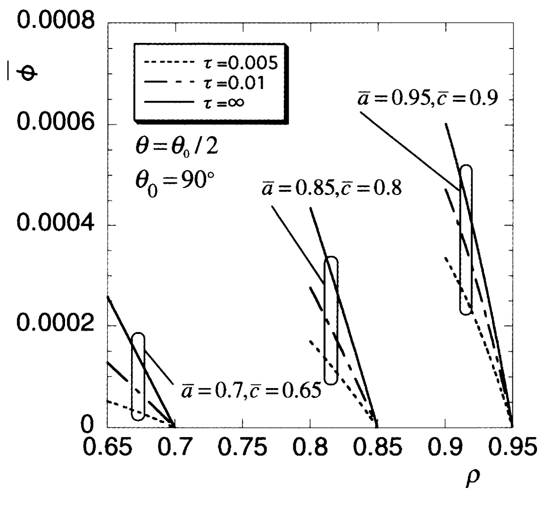

Fig. 7. Variation of the electric potential in the radial direction ($\theta = \theta_0/2$).

the thermal displacement $\bar{u}_r$ at the midpoint of the panel ($\theta = \theta_0/2$) is shown in Fig. 6(b). As shown in Fig. 6, it can be seen that the values for $\bar{a} = 0.95$ are considerably larger than ones for $\bar{a} = 0.7$ and $\bar{a} = 0.85$. And the variations with time vary depending on the radial ratio $\bar{a}$. Fig. 7 shows the variation of the electric potential at the midpoint of the panel. From Fig. 7, the values of electric potential increase when the radial ratio $\bar{a}$ increases.

The numerical results were obtained under the condition that the upper limits of series with respect to k and n are taken as $k = 16$ and $n = 130$, and the maximum of eigenvalue μ_j is 100.

4. Conclusions

In the present article, we obtained the exact solution for the transient temperature and transient piezothermoelastic response of a simply supported cylindrical composite panel composed of angle-ply laminate and piezoelectric material of crystal class mm2 due to a non-uniform heat supply in the circumferential direction. As an illustration, we carried out numerical calculations for the angle-ply laminated cylindrical panel composed of alumina fiber reinforced aluminum composite, associated with a piezoelectric layer of a cadmium selenide solid and examined the behaviors in the transient state for temperature change, displacement, stress, and electric potential distributions. Though numerical calculation were carried out for a two-layered anti-symmetric angle-ply laminated cylindrical panel associated with a piezoelectric layer, numerical calculation for hybrid laminated cylindrical panel with an arbitrary number of layer and arbitrary fiber orientation angles, associated with a piezoelectric layer can be carried out. Moreover, we conclude that we can evaluate not only all stress components of the combined cylindrical panel, but also the electric field of the piezoelectric layer quantitatively in a transient state.

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